A methodology for quality assessment in tensor images

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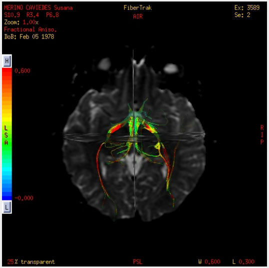
Introduction



- Increasing use of tensors in image processing due to new modalities:
  - Diffusion Tensor Imaging: usually, 3×3 symmetric semidefinite positive.
  - Stress Tensor: 3×3 symmetric.
  - Strain Tensor: 3×3 symmetric.
- New processing algorithms to deal with tensor images.
- How is the performance of these algorithms evaluated?.
  - Quantitatively
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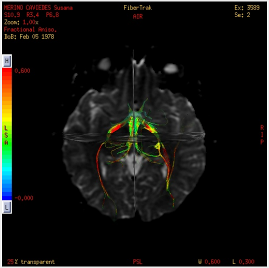

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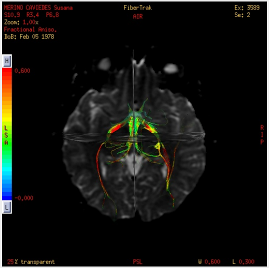

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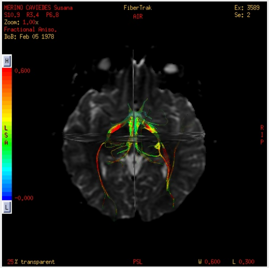

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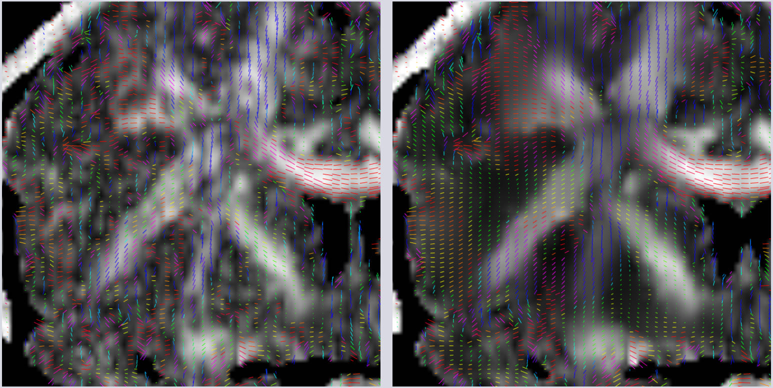

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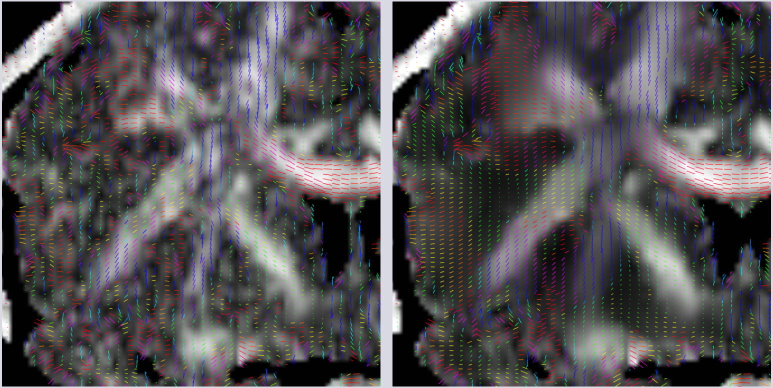
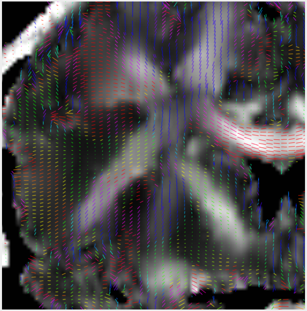


How to *measure* the algorithm performance?

Introduction (2)



- Quantitative evaluation:
  - Compute a scalar image from the tensor image and compute conventional quality measures.
  - Part of the tensor information is ignored.
- Qualitative evaluation:
  - Visual inspection.
  - Subjective evaluation.
  - Scalar images are used to visualize the tensor data. Part of the tensor information is ignored

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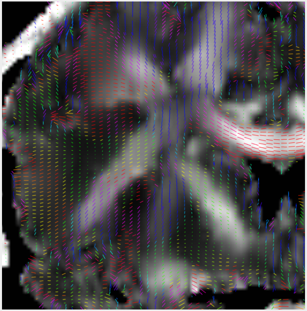

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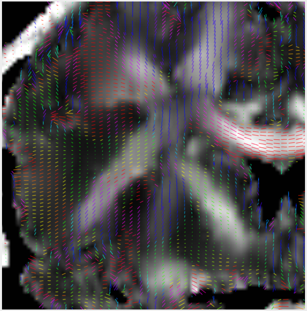

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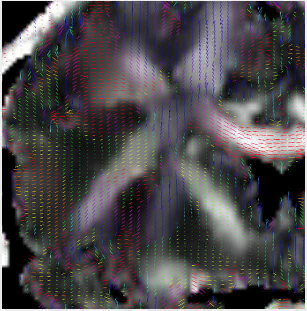

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A quantitave measure that takes into account all the tensor information is needed

Quality Measures for Scalar Images

Which is ‘better’?

Quality Assessment

Which is ‘better’?

How to *measure* the image quality?

Quality Assessment

- Error based measures: MSE, PMSE
- Structural based measures vs. pointwise measures.

SSIM

3 levels of comparison: Luminance, contrast and structural.

$$\text{SSIM}(I,J)(\mathbf{x}) = \frac{(2\mu_I(\mathbf{x})\mu_J(\mathbf{x}) + C_1)(2\sigma_{IJ}(\mathbf{x}) + C_2)}{(\mu_I(\mathbf{x})^2 + \mu_J^2 + C_1)(\sigma_I(\mathbf{x})^2 + \sigma_J(\mathbf{x})^2 + C_2)}$$

QILV

Based on local variance distribution

$$\text{QILV}(I,J) = \frac{2\mu_{V_I}\mu_{V_J}}{\mu_{V_I}^2 + \mu_{V_J}^2} \cdot \frac{2\sigma_{V_I}\sigma_{V_J}}{\sigma_{V_I}^2 + \sigma_{V_J}^2} \cdot \frac{\sigma_{V_I V_J}}{\sigma_{V_I}\sigma_{V_J}}$$

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Main idea: extend Image quality assessment indexes to Tensor field quality assessment

Tensor Image Quality Assessment

Quality assessment

- Usually MSE: mean value of the Frobenius norm of the difference between tensors; mean Euclidean distance (MED).
- Every tensor components should be taken into account.
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- Statistics of the tensor images are required. How are tensor statistics computed?

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Statistics of tensor data:

Special features of tensors that should be taken into account:

- Suppose the tensor **T** describes a transformation, and $\mathbf{T}^{-1}$ is the inverse transformation. Their composition $\mathbf{T}^{-1}\mathbf{T}$ is the identity tensor. Their mean should be the identity tensor–> Geometric means are required.
- Swelling effect. The determinant of the mean tensor can be higher than the determinant of the individual tensor if Euclidean mean is computed.
- In some cases, tensor definite or semidefinite positiveness constraints should be preserve-> Riemmanian metrics that avoid negative eigenvalues are used.

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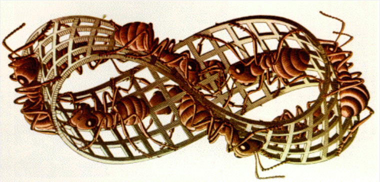
Framework

- Some frameworks to compute statistics of tensor images have been defined.
- LogEuclidean framework:
  - The logarithm of the tensor is computed and their components are arranged in a vector.
  - Statistics are computed in the vector space using Euclidean metric.
  - Return to the original space by means of the exponential.



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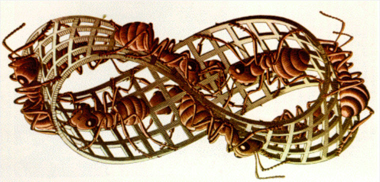

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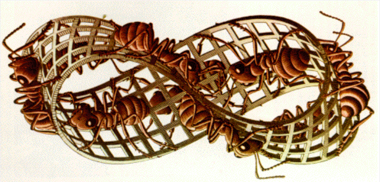

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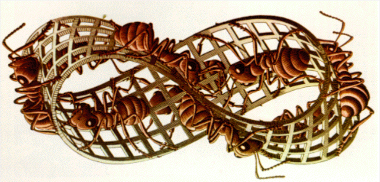

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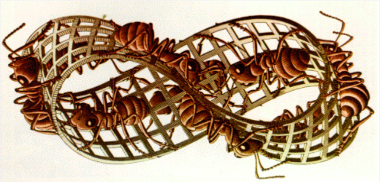

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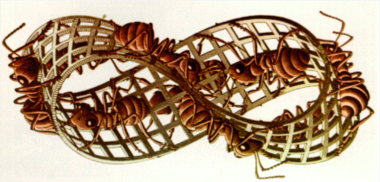

Tensor Image Quality Assessment

Methodology for quality assessment

1. Translate tensors to the LE domain.
2. Compute the required statistics of the tensor image in the LE domain.
3. Compute scalar quality indexes for each LE-vector component–> Quality index vector.
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Experiments

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Measures

Three measures are compared:

- Pointwise Mean Euclidean Distance (MED)
- Tensor-adapted QILV.
- Tensor-adapted MSSIM.

Synthetic Tensor Field

- Built for the experiments.
- The golden standard: 2D, 128×128.
- Built considering DTI and Stejskal-Tanner equation.
- Degradation over DWI

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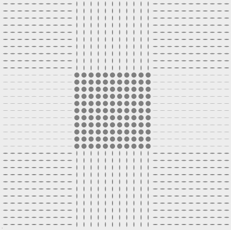
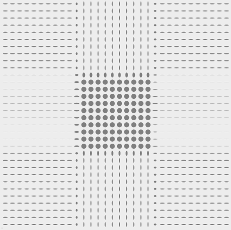
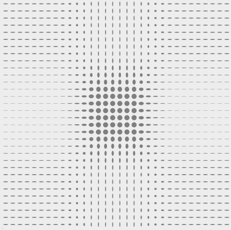


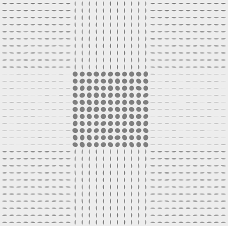
I



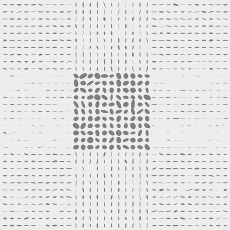
Blur : $I_{5\times5}$



Blur: $I_{21\times21}$

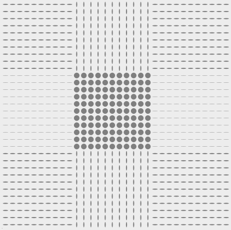


Noise: $I_{\sigma=20}$

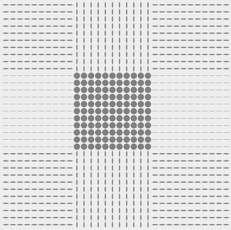


Noise: $I_{\sigma=80}$

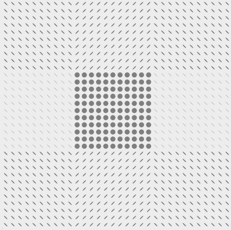
Experiments



$I_{10\%}$



$I_{25\%}$



$I_{\pi/4}$

Experiments

	$I_{5\times5}$	$I_{9\times9}$	$I_{15\times15}$	$I_{21\times21}$	$I_{\sigma=20}$	$I_{\sigma=40}$	$I_{\sigma=60}$	$I_{\sigma=80}$
MSSIM	0.90	0.84	0.82	0.80	0.36	0.31	0.29	0.27
QILV	0.72	0.71	0.70	0.67	0.70	0.59	0.57	0.53
MED($\times10^{-6}$)	0.07	0.32	0.35	0.39	0.02	0.07	0.17	0.27

	$I_{10\%}$	$I_{25\%}$
MSSIM	0.99	0.91
QILV	0.99	0.92
MED($\times10^{-6}$)	0.07	0.41

QILV vs SSIM

- Noise has more influence in MSSIM.
- Blurring has more influence in QILV.
- The behaviour is similar to the scalar case.

Experiments

	$I_{5\times5}$	$I_{9\times9}$	$I_{15\times15}$	$I_{21\times21}$	$I_{\sigma=20}$	$I_{\sigma=40}$	$I_{\sigma=60}$	$I_{\sigma=80}$
MSSIM	0.90	0.84	0.82	0.80	0.36	0.31	0.29	0.27
QILV	0.72	0.71	0.70	0.67	0.70	0.59	0.57	0.53
MED($\times10^{-6}$)	0.07	0.32	0.35	0.39	0.02	0.07	0.17	0.27

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Structural vs. Pointwise

- MED is high although the structure remains similar.
- Structural measures are few influenced by changes in the tensor size if structure is preserved.

Tensor based vs Scalar based

Original field is compared with reoriented tensors $\tau \in [0, \pi]$.

- MSSIM and QILV computed over FA: The value is constant for every rotation angles.
- MSSIM and QILV over tensor: varies with the rotation angle; they take into account the tensor orientation
- $\tau = \frac{\pi}{16}$, MSSIM= 0.78, QILV=0.67
- $\tau = \frac{\pi}{4}$, MSSIM=0.71, QILV=0.56.

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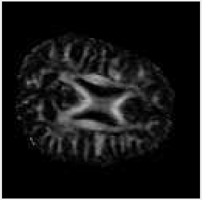
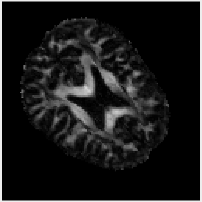
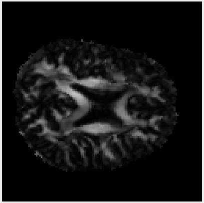


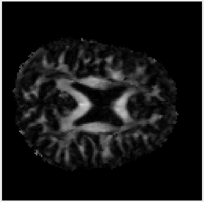
Fixed image



Moving image



Registered without reorientation



Registered with reorientation

	Moving	Reg. without reorient	Reg. with reorient
Tensor MSSIM	0.4566	0.6549	0.8485
Tensor QILV	0.2624	0.4514	0.6363
MED	0.1588	0.0652	0.0457
FA MSSIM	0.5742	0.9700	0.9700
FA QILV	0.4865	0.8710	0.8710

Conclusions

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- A methodology to extend quality measures to tensor data is proposed.
  - Considers every tensor component.
  - Allows the extension of structural based measures.
- The behaviour of tensor adapted measures for tensor images is similar to the behaviour of the original quality measures for scalar images.
- Tensor adapted measures are required to correctly evaluate the performance of algorithm that deals with tensor images.
- More specific measures could be defined for specific image modalities.

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Thanks for your attention