Noise Estimation in Magnetic Resonance SENSE Reconstructed Data

S. Aja-Fernández, G. Vegas-Sanchez-Ferrero,
R. de Luis-García, A. Tristán-Vega

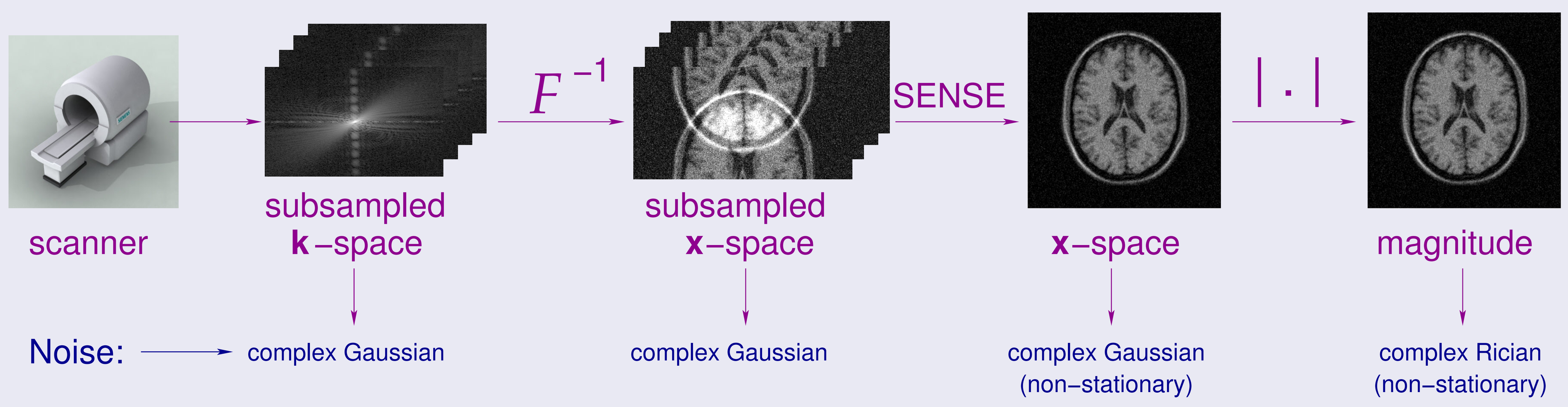
Laboratorio de Procesado de Imagen. Universidad de Valladolid. Spain

Universidad de Valladolid

Laboratorio de Procesado de Imagen

Parallel imaging methods allow to increase the acquisition rate via subsampled acquisitions of the k-space. SENSE reconstruction process yields to a variance of noise value which is dependent on the position within the image. Hence, the traditional noise estimation methods based on a single noise level for the whole image fail. Accordingly, we propose a novel method to recover the complete spatial pattern of the variance of noise in SENSE reconstructed images up from the sensitivity maps of each receiver coil. Our method fits applications in statistical image processing tasks such as image denoising.

Statistical Noise Model in SENSE



- Subsampled multi coil MR data reconstructed with Cartesian SENSE follows a Rician distribution at each point of the image.
- The resulting distribution is non-stationary: the variance of noise will vary from point to point across the image.
- The final value of the variance of noise at each point will only depend on the covariance matrix of the original data and on the sensitivity map, and not on the data themselves.

Signal in complex reconstructed signal is non stationary Gaussian, with variance:

$$\sigma_i^2 = \mathbf{W}_i^*(x,y)\,\Sigma\,\mathbf{W}_i(x,y).$$

with $\mathbf{W}_i^*(x,y)$ derived from the sensitivity maps in each coil, $\mathbf{C}_l(x,y)$:

$$\mathbf{W}(x,y) = (\mathbf{C}^*(x,y)\mathbf{C}(x,y))^{-1}\mathbf{C}^*(x,y).$$

Noise Estimation in SENSE

Background of the SENSE reconstructed image is a (non-stationary) Rayleigh, therefore

$$\mathrm{E}\{M^2(\mathbf{x})\} = 2\cdot\sigma_R^2(\mathbf{x}).$$

We assume the following covariance matrix:

$$\Sigma = \sigma_n^2\begin{pmatrix} 1 & \rho^2 & \cdots & \rho^2 \\ \rho^2 & 1 & \cdots & \rho^2 \\ \vdots & \vdots & \ddots & \vdots \\ \rho^2 & \rho^2 & \cdots & 1 \end{pmatrix} = \sigma_n^2\left(\mathbf{I}+\rho^2[\mathbf{1}-\mathbf{I}]\right).$$

For each $\mathbf{x}$ value, we define the global map

$$\mathscr{G}_{W_i} = \mathbf{W}_i^*\left(\mathbf{I}+\rho^2[\mathbf{1}-\mathbf{I}]\right)\mathbf{W}_i,\quad i=1,\cdots,r$$

Global map $\mathscr{G}_W(\mathbf{x})$ can be easily inferred from the $\mathscr{G}_{W_i}$ values. Then

$$\sigma_n^2 = \frac{\mathrm{E}\{M^2(\mathbf{x})\}}{2\,\mathscr{G}_W(\mathbf{x})}$$

By using this regularization, we can assure a single σ_n^2 value for all the points in the image. Following the estimation philosophy in [1,2] we define a noise estimator based on the sample second order moment $\langle M^2(\mathbf{x})\rangle_{\mathbf{x}}$ (Gamma distributed) and then

$$\mathrm{mode}\left\{\frac{\langle M_L^2\rangle_{\mathbf{x}}}{\mathscr{G}_W(\mathbf{x})}\right\} = 2\sigma_n^2\frac{|\eta(\mathbf{x})|-1}{|\eta(\mathbf{x})|} \approx 2\sigma_n^2$$

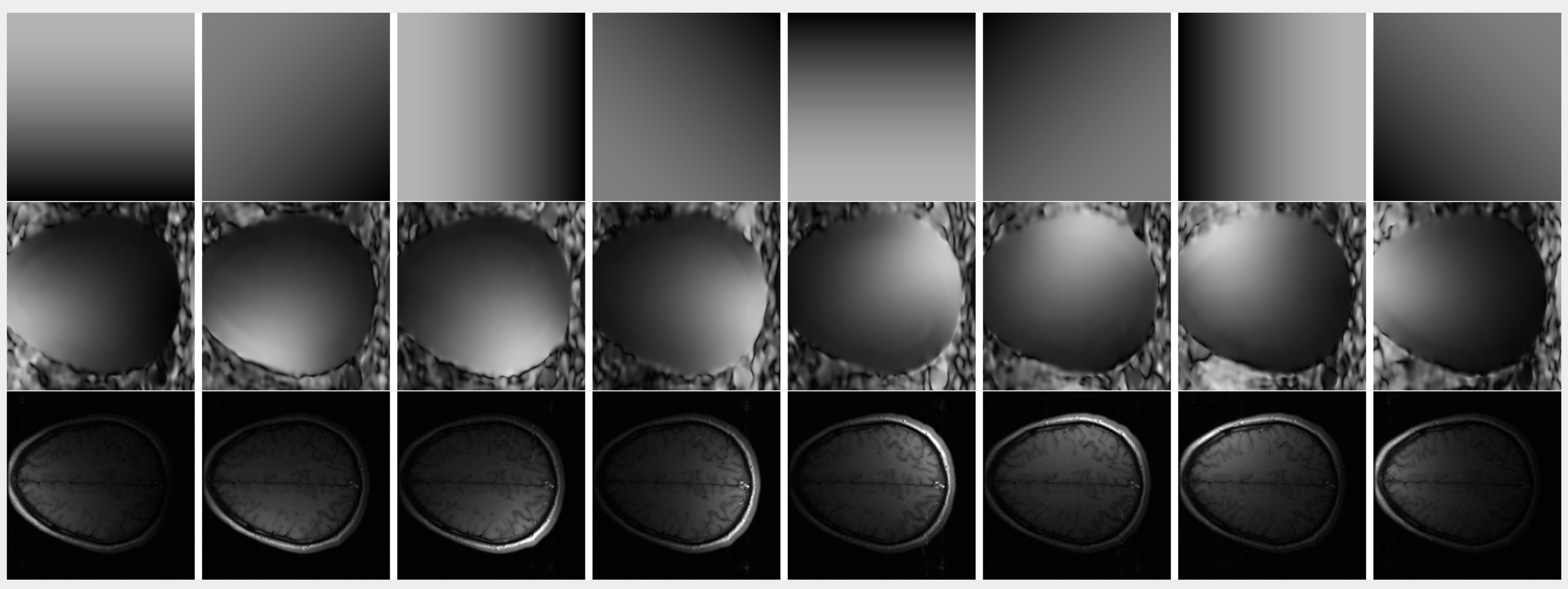
The estimator is then defined as

$$\widehat{\sigma_n^2} = \frac{1}{2}\mathrm{mode}\left\{\frac{\langle M_L^2(\mathbf{x})\rangle_{\mathbf{x}}}{\mathscr{G}_W(\mathbf{x})}\right\}$$

and noise in each pixel is estimated as

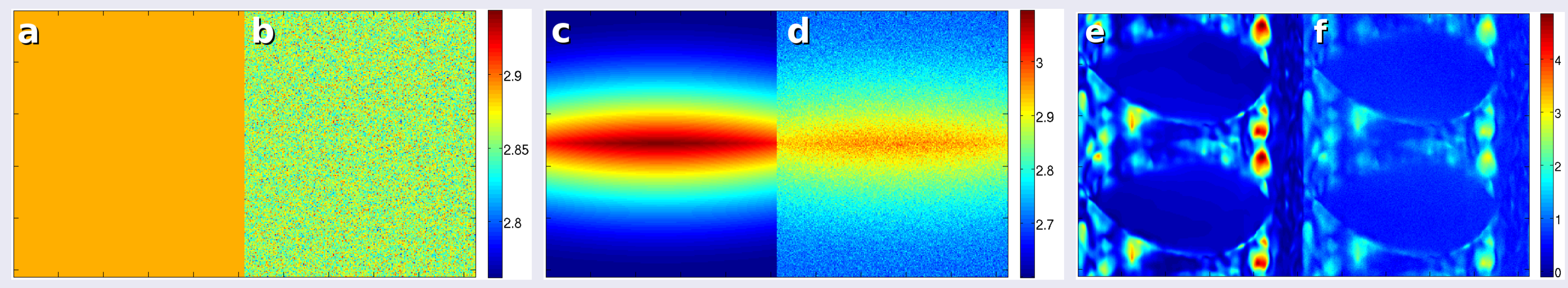
$$\widehat{\sigma_R^2}(\mathbf{x}) = \frac{1}{2}\mathrm{mode}\left\{\frac{\langle M_L^2(\mathbf{x})\rangle_{\mathbf{x}}}{\mathscr{G}_W(\mathbf{x})}\right\}\mathscr{G}_W(\mathbf{x})$$

This estimator is only valid over the background pixels. no segmentation of these pixels is needed: the use of the mode operator allows us to work with the whole image.



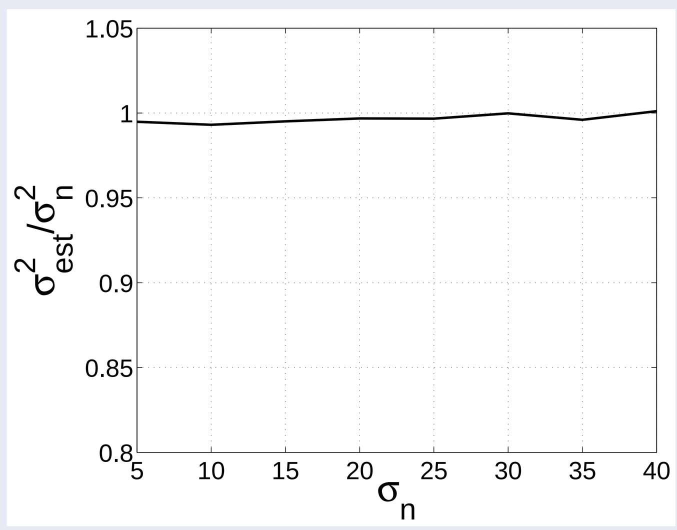
Experiments and results

1.-Variation of noise: test of the variation of parameter $\sigma_R^2(\mathbf{x})$ across the image in SENSE. 2 sets of sensitivity maps, $\sigma_l^2 = 1$ and $\rho^2 = [0, 0.1]$. Estimation is done over 5000 samples. When correlations are taken into account, even using the same synthetic sensitivity map, the standard deviation of noise of the reconstructed data is not the same for every pixel: the noise is no longer spatial-stationary.

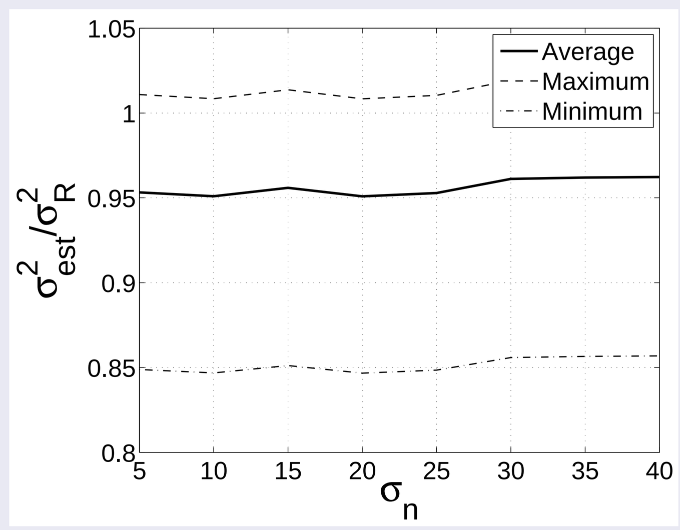


Maps of $\sigma_R^2(\mathbf{x})$ in the final image: (a-c-e): Theoretical values. (b-d-f): Estimated from samples. (a-b) Synthetic Sensitivity Map with no correlation. (c-d) Synthetic Sensitivity Map with correlation between coils. (e-f) Real sensitivity map with correlation between coils.

2.-Synthetic Experiment: 2D slice from BrainWeb, intensity values in $[0-255]$ (Averages: White Matter 158, Gray Matter 105, cerebrospinal fluid 36, the background 0). 8-coil system is simulated using an artificial sensitivity map. Each coil corrupted with Gaussian noise with $\sigma_n \in [5-40]$. The k-space is uniformly subsampled by a factor of 2 and reconstructed with SENSE. The average of 100 experiments is considered.



(a) With known $\mathscr{G}_W(\mathbf{x})$



(b) Blind estimation

3.-Real data: T1 acquisition done in a GE Signa 1.5T EXCITE, FSE pulse sequence, 8 coils, TR=500msec, TE=13.8msec, 256×256 and FOV: 20cm$\times$20cm. σ_n is estimated from the Gaussian complex data:

Real component	$\widehat{\sigma_n} = 4.1709$
Imag. component	$\widehat{\sigma_n} = 4.0845$

A subsampled acquisition is simulated and reconstructed with SENSE. σ_n is first estimated assuming the map $\mathscr{G}_W(\mathbf{x})$ known and unknown:

Magnitude ($\mathscr{G}_W(\mathbf{x})$ known)	$\widehat{\sigma_n}/\sqrt{r} = 4.1728$
Magnitude ($\mathscr{G}_W(\mathbf{x})$ unknown)	$\widehat{\sigma_n}/\sqrt{r} = 4.8404$

Conclusions

- The stationarity assumption is no longer valid when parallel imaging and SENSE reconstruction are considered. The variance of noise becomes $\mathbf{x}$-dependent.
- The estimation of the spatially variant $\sigma_R^2(\mathbf{x})$ allows us to re-use many of the methods in literature proposed for single-coil Rician models.
- The estimation method proposed shows to be accurate, robust and easy to use.
- Limitations: (1) correlation between coils must be known beforehand, as well as the sensitivity map; (2) Some post processing software in the scanner may add a mask to data, which eliminates part of the background, drastically reducing the number of points available for noise estimation.

Acknowledgements

This work was partially supported by CDTI (cvREMOD, ref. CEN-20091044), Junta de Castilla y León (VA0339A10-2, SAN103/ VA40/11, GRS 555/A/10) and MICIN (TEC2010-17982). Authors acknowledge Quirón (Valencia) and J. Sanchez Gonzalez for the collaboration in cardiac sequence acquisition.

References

[1] S. Aja-Fernández, C. Alberola-López, and C.-F. Westin, "Noise and signal estimation in magnitude MRI and Rician distributed images: A LMMSE approach," *IEEE Trans. Image Proccess.*, vol. 17, no. 8, pp. 1383–1398, Aug. 2008.

[2] S. Aja-Fernández, A. Tristán-Vega, and C. Alberola-López, "Noise estimation in single and multiple coil MR data based on statistical models," *Magn. Reson. Imag.*, vol. 27, pp. 1397–1409, 2009.

http://www.lpi.tel.uva.es/~santi

sanaja@tel.uva.es