DWI acquisition schemes and Diffusion Tensor estimation

A simulation–based study

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- Diffusion Weighted Magnetic Resonance Imaging (DW-MRI) well-known established imaging modality used to measure water diffusivity in tissues.
- Based on Diffusion tensor (DT): Least Squares (LS) *de facto* standard to estimate DT.
- [Salvador05] and [Tristan09] theoretical framework to estimate DT from single- and multiple-coil systems using Weighted LS.
- Simulation-based study of the effect of the acquisition schemes over the estimated DT. Four cases: single coil acquisition, multiple coil acquisition (no subsampling) and subsampled multiple coil acquisition with pMRI reconstruction (SENSE and GRAPPA).

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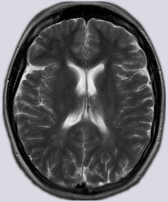
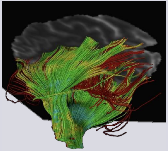
Diffusion Tensor MRI

MRI *versus* dMRI

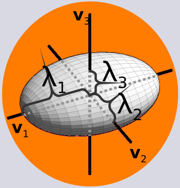


MRI: distinguish tissues



dMRI: track nerve fibers

Main idea behind Diffusion Tensor MRI



- The white matter is strongly anisotropic
- This anisotropy can be indirectly characterized through the diffusion of water molecules
- A diffusion tensor is assigned to each 3D pixel :
  - Rank-2 tensor $\rightarrow 3 \times 3$ symmetric matrix
  - Eigenvectors: principal diffusion directions
  - Eigenvalues: amount of diffusion along them

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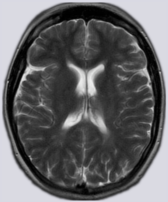

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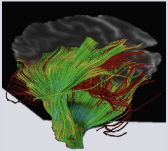

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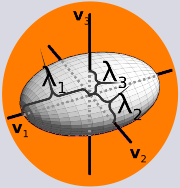

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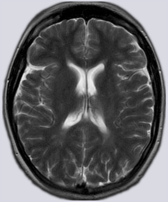

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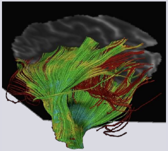

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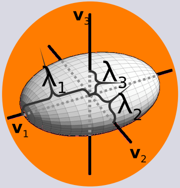

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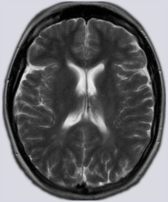

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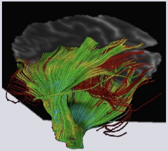

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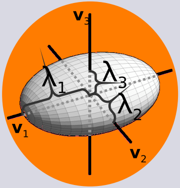

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Interpretation of DT-MRI data

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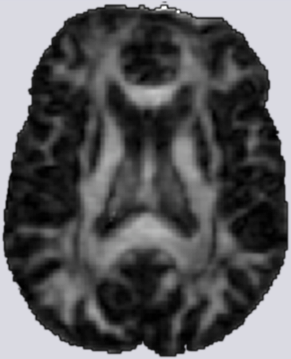

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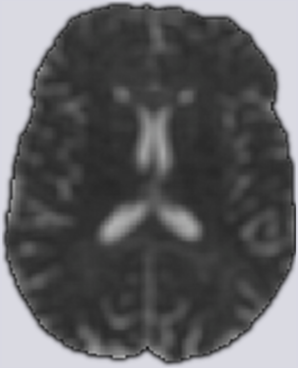

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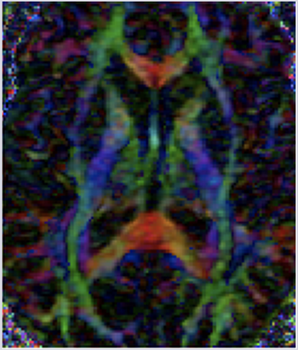

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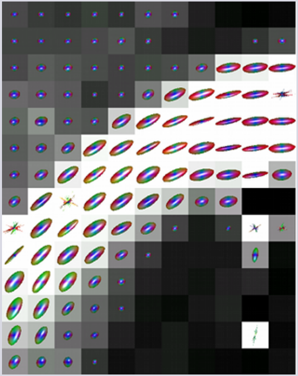

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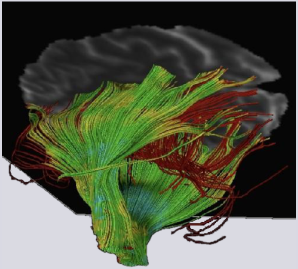

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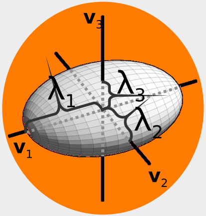

- Most dMRI applications based on single tensor approach.
- Known weakness of single tensor: fiber crossing or kissing.
- Least Squares (LS) techniques *de facto* standard: Weighted Least Squares (WLS).
- The big question: To use more gradient directions or more repetitions (NEX)?
- In [Tristan09]: variance in the estimation reduced by increasing the number of gradient directions. Bias reduced by increasing NEX.
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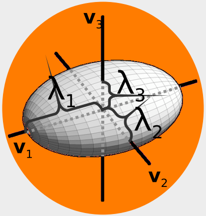

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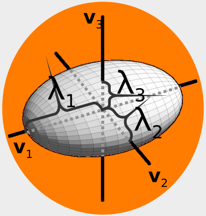

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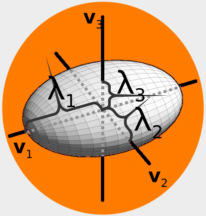

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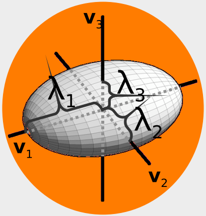

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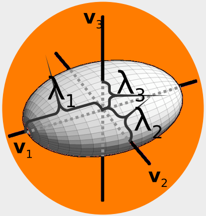

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Signal and noise statistical models in MR

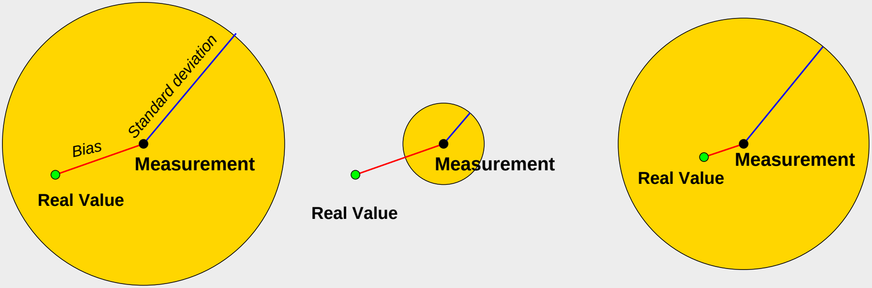
Number of coils	**Adquisition**	**Statistical Model**	**Stat. model of the background**
1 coil	Single coil	Rician (Stationary)	Rayleigh
Multiple coils	No subsampling+ SoS	Non-central Chi (Non-Stationary)	Central Chi
Multiple coils	pMRI+ SENSE	Rician (Non-Stationary)	Rayleigh
Multiple coils	pMRI+ GRAPPA+ SoS	Non-central Chi (Non-Stationary, effective parameters)	Central Chi

Tensor fitting based on Weighted Least Squares

Estimation error for a simplified scenario [Tristan09]

The error (**MSE**) for multiple-coil is defined as

$$\mathrm{MSE} \simeq \underbrace{\left[\frac{K_1}{N}\left(\frac{1}{\mathrm{SNR}^2} - \frac{1}{\mathrm{SNR}^4}(3L-4)\right)\right]}_{\mathrm{Var(estimation)}} + \underbrace{\left[\frac{1}{\mathrm{SNR}^4} 3(L-1)^2\right]}_{\mathrm{bias}^2\mathrm{(estimation)}}$$



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Hypotheses about the acquistion modalities

1. The contribution of the bias will be much smaller for the Rician than for the nc-χ case. Accordingly, the bias on the estimation will be smaller in one-coil systems and in pMRI when reconstructed with SENSE.
2. The error due to the variance will be reduced by increasing the number of gradients (N). However, this is not the case for the bias, which cannot be reduced from taking more gradients. The only way to do so is by increasing the SNR. This can be done by properly filtering the data or by increasing the NEX (number of repetitions in the scanner).
3. pMRI reconstructed images are known to decrease the SNR according to the so-called *g-factor*. So, the variance and bias of the error are expected to grow in these cases when compared to the fully-sampled multiple coil case.
4. Finally, equation does not take into account one additional source of error: in subsampled pMRI the missing data must be estimated, i.e. the spatial aliasing must be corrected. The error in this estimation will also propagate to the DT estimation.

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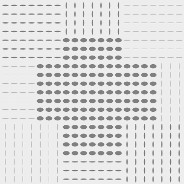
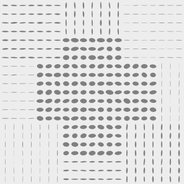
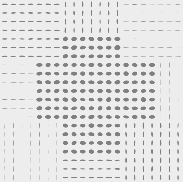


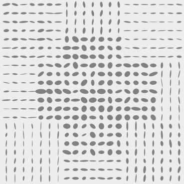
(a) Original



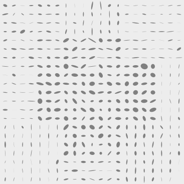
(b) Rician



(c) pMRI-SoS



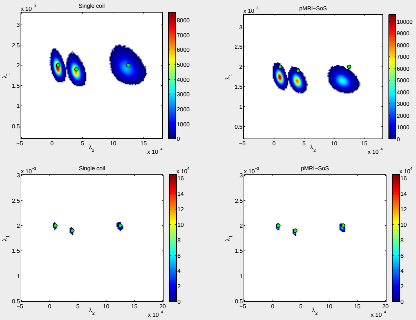
(d) SENSE



(e) GRAPPA

Synthetic 2D Tensor Field 2D used for the first experiment. Original size 128×128. Original and noisy cases, 8 coils, 5 gradients and $\sigma_n = 35$.

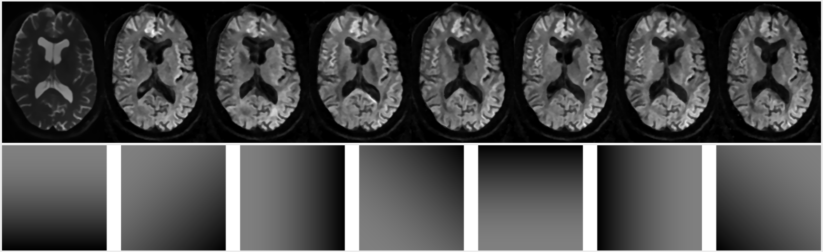
Synthetic experiments



(a) Rician (b) pMRI-SoS (c) SENSE (d) GRAPPA

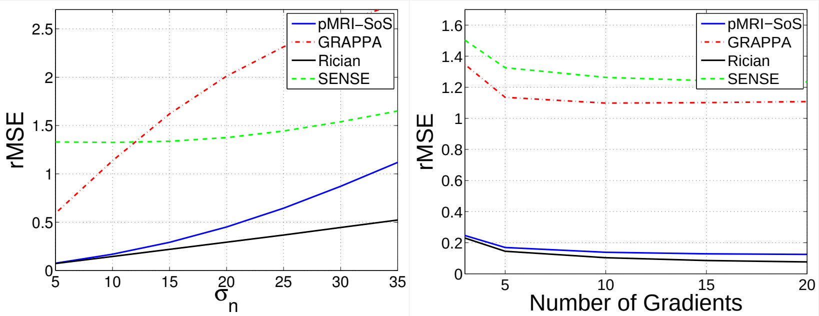
Two dimensional histograms of the distribution of the estimated eigenvalues: λ_1 vs. λ_2. Top row: $\sigma_n = 35$ and 5 gradient directions. Low row: $\sigma_n = 10$ and 15 gradient directions. In green the original eigenvalues.

Realistic DWI phantom



A realistic DWI phantom is used, [Tristan09b]. A $256 \times 256 \times 81$ volume, spatial resolution of $1\text{mm} \times 1\text{mm} \times 1.7\text{mm}$, 15 gradient directions and 1 baseline.

[Tristan09b] A. Tristán-Vega and S. Aja-Fernández, “Design and construction of a realistic DWI phantom for filtering performance assessment,” in *MICCAI 2009*, 2009.



rMSE of the tensor estimation for (Left) different σ_n values (and 5 gradients); (Right) different number of gradients (and $\sigma_n = 10$)

$$\mathrm{rMSE}(x) = \frac{\sqrt{(\widehat{\lambda_1}(x) - \lambda_1(x))^2 + (\widehat{\lambda_2}(x) - \lambda_2(x))^2 + (\widehat{\lambda_3}(x) - \lambda_3(x))^2}}{\lambda_1(x)}$$

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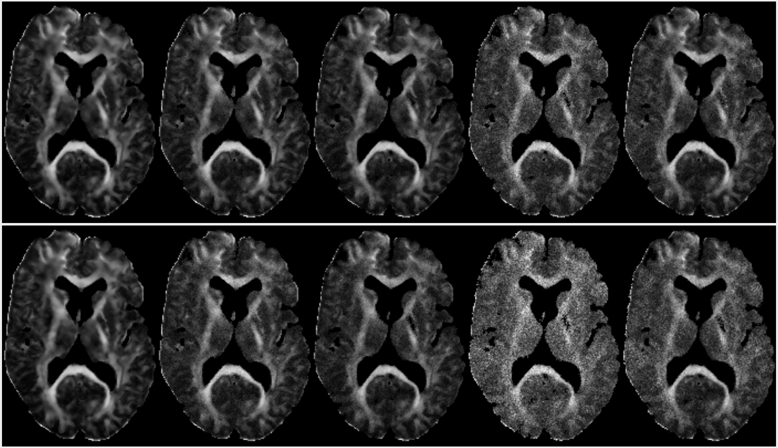

Fractional Anisotropy. From left to right: Original non-noisy data; Rician case; pMRI-SoS case; SENSE case; GRAPPA case. Top row $\sigma_n = 10$ (average SNR in gray matter in the gradient images 40). Low row: $\sigma_n = 35$ (average SNR in gray matter in the gradient images 11.4).

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Conclusions

- The methods considered can be modeled or approximated by a nc-χ distribution (Rician being a particular case).
- Three variables must be taken into account: the **number of coils** (that mainly affects to the bias in the error), the **number of gradients** (that mainly affects to the variance of the estimation error) and the **SNR** that globally affects the estimation error.
- Methods using multiple coils show a greater bias.
- pMRI: accelerate the acquisition process but worsening the SNR: greater variance and bias in DT estimation.

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A simulation–based study

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