Noise estimation and diffusion signal reconstruction: From cradle to parallel imaging

What type of noise ‘infects’ the data and by filtering it out are we (black) magically creating something new?

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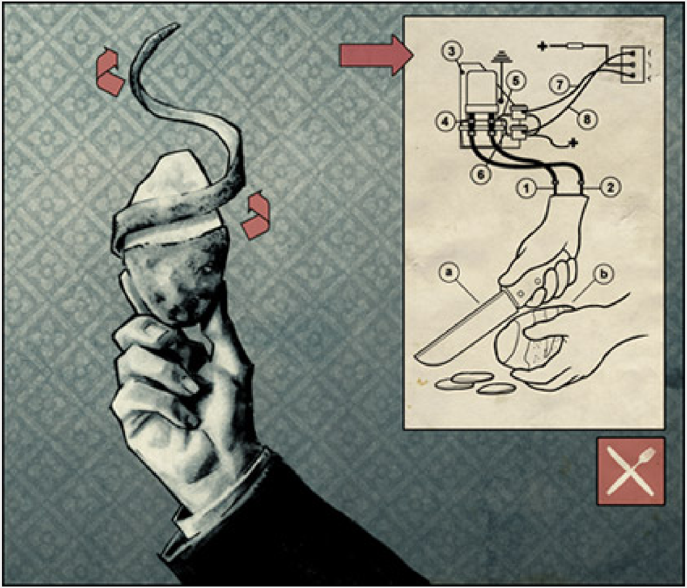
Illustrations: David Aja

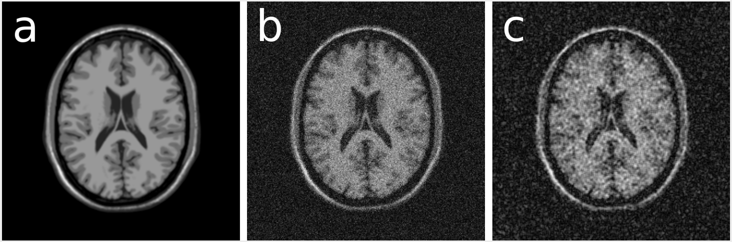
Outline

1. Introduction
2. Signal and Noise statistical models
3. Noise filtering and signal estimation
4. Noise estimation
5. Effects on dMRI
6. Pitfalls and conclusions

Thinking about the problem



Noise in MR data: An aesthetic problem?



Diffusion tensor field over FA

- Noise is one of the main sources of quality deterioration in magnetic resonance (MR) data.
- Is noise just a problem for “image quality" and visual inspection?
- Affecting: segmentation, registration, tensor estimation...
- In dMRI: noise and filtering may affect the estimation of direction and amount of diffusion.

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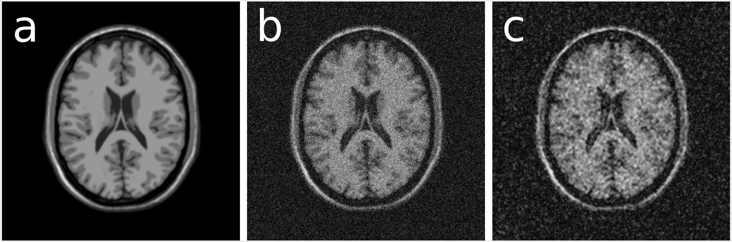

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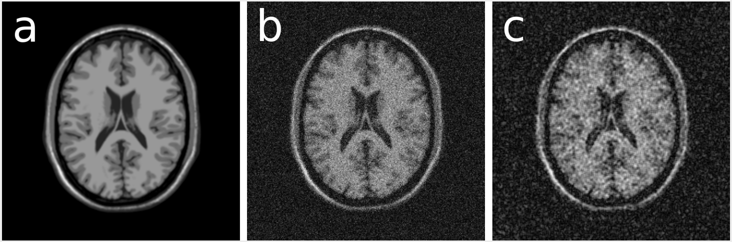

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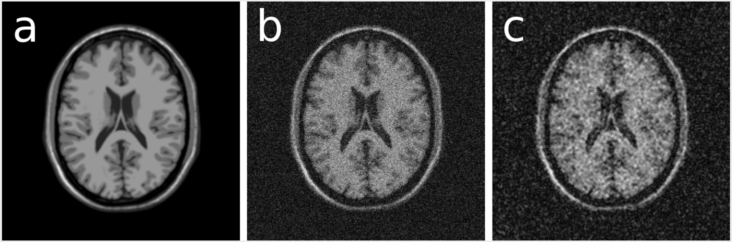

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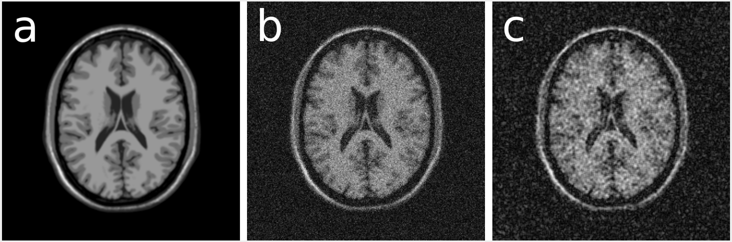

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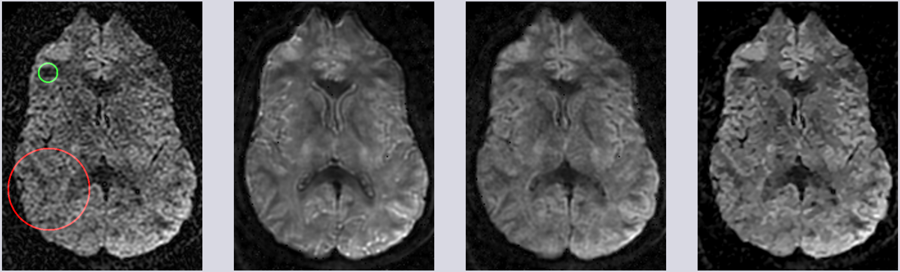
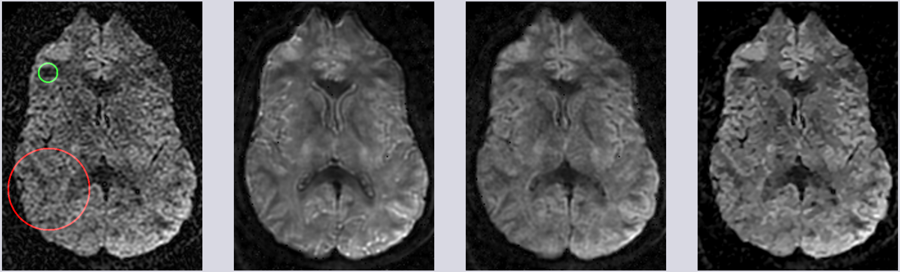
MR filtering



Original J-LMMSE LMMSE 15 UNLM

MR filtering



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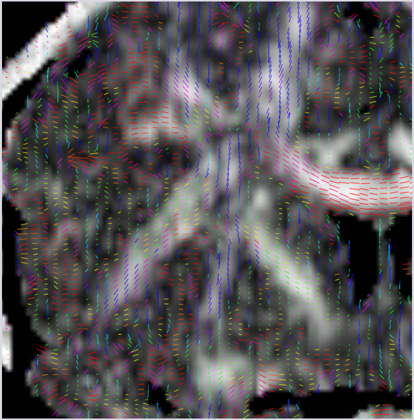
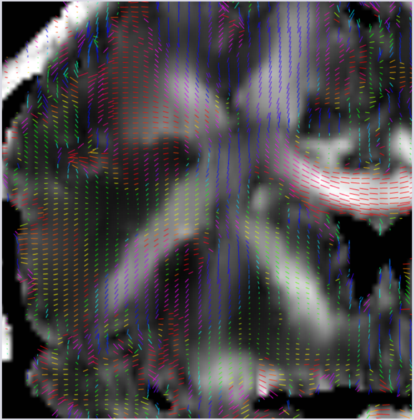
We can *clean* the images... is it enough in dMRI?

Diffusion Tensor, real example

Diffusion tensor field over FA



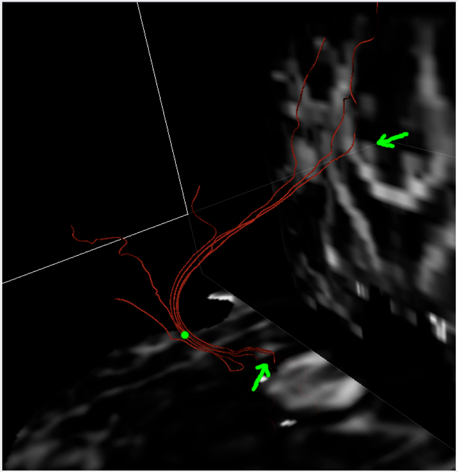
Without filtering

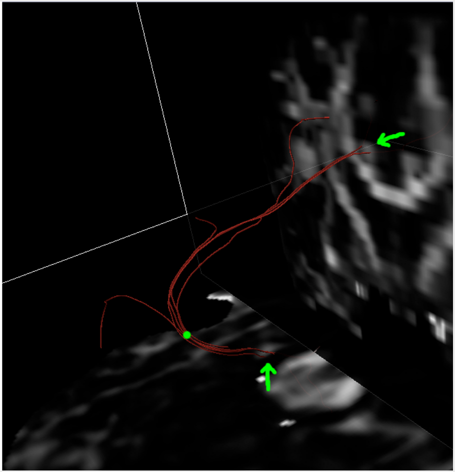


LMMSE filtered

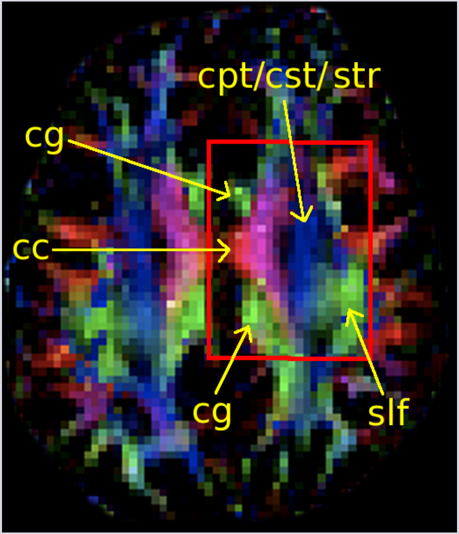
Aja-Fernández et al., Oct. 2008. Restoration of DWI data using a Rician LMMSE estimator. IEEE Trans. Med. Imaging 27 (10).

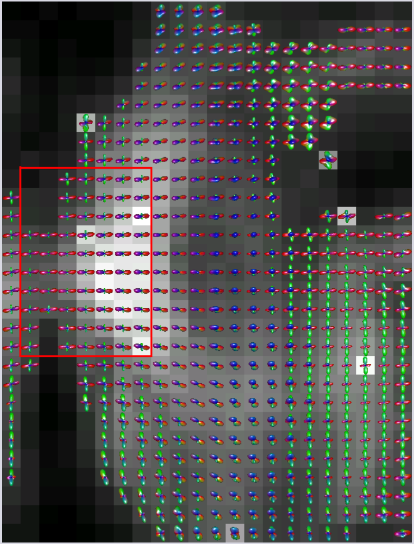
Diffusion Tensor, real example





Q-Balls imaging, real example





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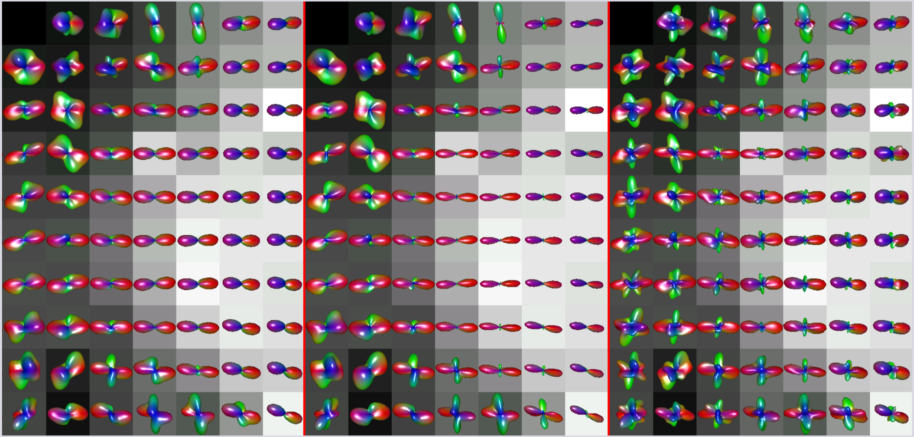
Comparison: Q-Balls, DOT, OPDT



Without LMMSE-*N* filtering

Q-Balls imaging, real example

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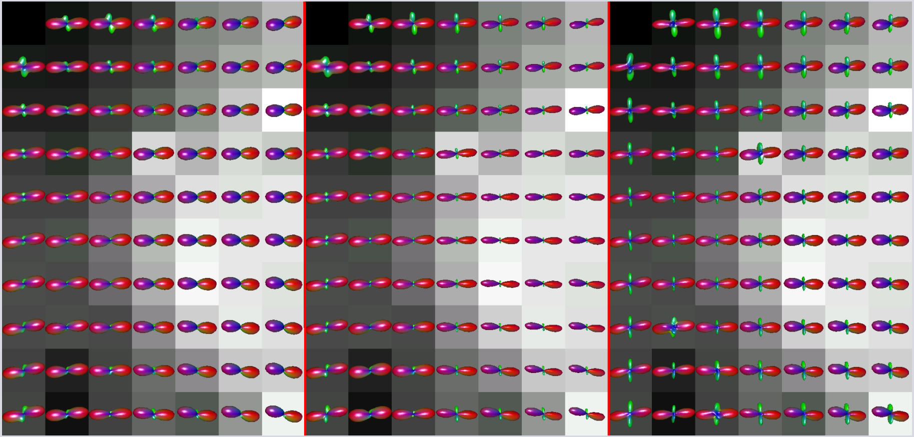

With LMMSE-*N* filtering

Overview

- Noise is known to be one of the main sources of quality deterioration in magnetic resonance (MR) data.
- We want to get rid of that noise but preserving the underlying structures (very important in dMRI).
- Accordingly:
  1. Filtering methods based on data structure and modeling of noise behavior. Bayesian and probabilistic modeling.
  2. Quality assessment methods to test the goodness of proposed algorithms.
  3. Estimation of parameters out of data: variance of noise estimation.
  4. Filtering and preprocessing: model based.

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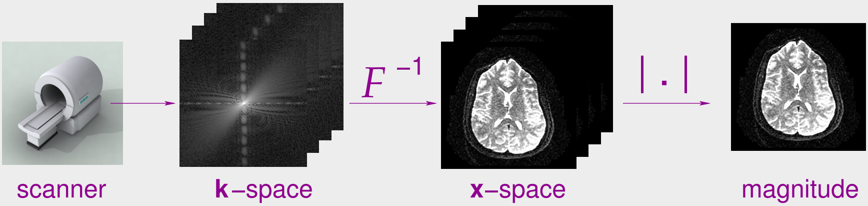

k-space

- Complex Gaussian noise
- Uncorrelated
- Stationary

x-space

- Complex Gaussian noise.
- Uncorrelated?
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Magnitude

- Magnitude of complex Gaussian: **Rician.**
- Uncorrelated?
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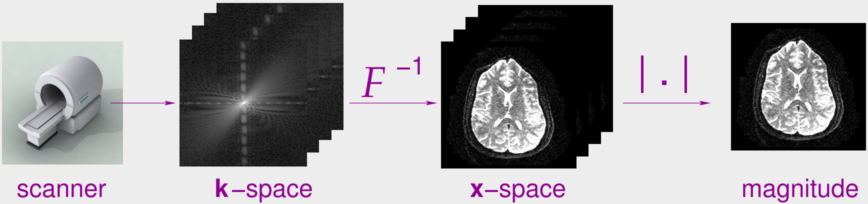

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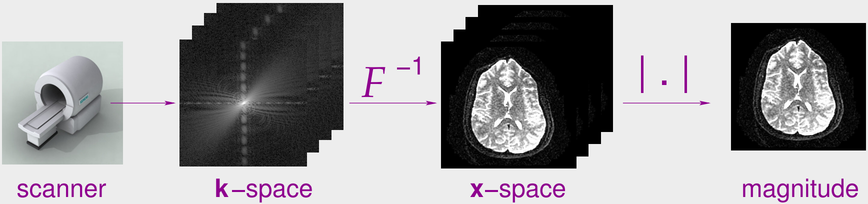

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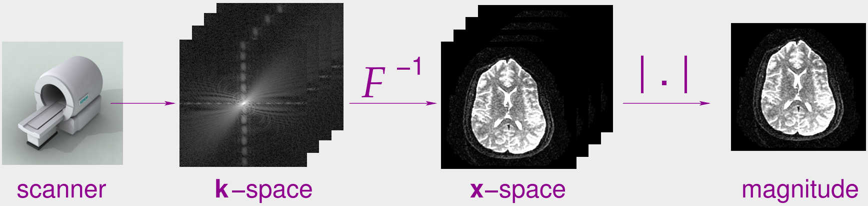

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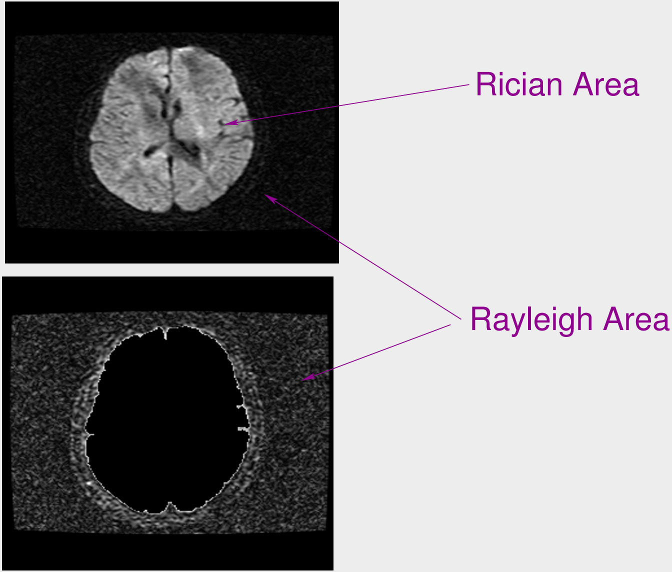
Areas in the image



Signal and noise statistical models in MR

Before Magnitude	
k-space	Complex Gaussian
x-space	Complex Gaussian

Composite Magnitude Image			
Number of coils	**Acquisition**	**Statistical Model**	**Stat. model of the background**
1 coil	Single coil	Rician (Stationary)	Rayleigh
Multiple coils	No subsampling+ SoS	Non-central Chi (Stationary)	Central Chi
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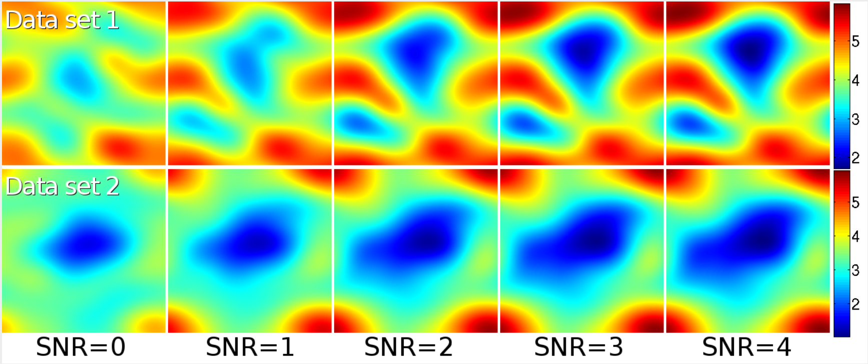
For high SNR: always possible to use **Gaussian** assumption.

Stationarity (brief, quick and intuitive)

Variance of noise

- Stationary: same σ_n^2 value for every pixel.
- Non-Stationary: σ_n^2 varies along the image.



Too many abstract concepts...

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Let’s go back to earth!

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The mambo jumbo: Eliminating the noise

- Purpose: eliminate the noise in MR data without destroying any signal information.
- Basically: we want to improve the SNR of our data
- In dMRI special attention to noise models: filtering may introduce bias.
- Trade off between denoising and structure keeping
- REMEMBER: We are not *inventing* data or *cleaning* an image; we are estimating a signal out of noisy data. Ideally: we are recovering the most likely or possible signal based on the data we have.

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- Rician Based: [Mcgibney93] (Conventional Approach), [basu2006], [Koay06]
- Bayesian Framework: [Marzetta95] (Expectation–Maximization, EM estimation of Rician signal) [Sijbers98], [Sijbers98c], [Sijbers04] (maximum likelihood, ML, signal estimation), [AjaIP08], [AjaTMI08], [AjaMiccai07], [TristanMiccai08], [Martinfernandez2007] (Linear Minimum Mean Squared Error, LMMSE, signal estimation).
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- Other methods: [Awate05] (Nonparametric Neighborhood Statistics), [Nowak99], [Pizurica03] (Wavelets).
- Filtering using all the DWIs information: [TristanMiccai08], [TristanFiltrado09] (LMMSE and NLM taking into account information of all DWIs).

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- Filtering using all the DWIs information: [TristanMiccai08], [TristanFiltrado09] (LMMSE and NLM taking into account information of all DWIs).

Main filtering approaches

	Method	Comments	Quality	Time
No model	Anisotropic Diffusion Averaging NLM	Biased Biased, smooth Biased	*** * ****	*** * ****
Rician	Conventional Approach LMMSE UNLM ORNR-AD	Smooth Simple Bias corrected Bias corrected	** *** **** ****	* * **** ***
DWI	LMMSE-N UNLM-N	Multiple DWI Multiple DWI	***** *****	** *****

Alternative: filtering in the complex domain (scanner) using Gaussian model.

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	Method	Comments	Quality	Time
No model	Anisotropic Diffusion	Biased	***	***
	Averaging	Biased, smooth	*	*
	NLM	Biased	****	****
Rician	Conventional Approach	Smooth	**	*
	LMMSE	Simple	***	*
	UNLM	Bias corrected	****	****
	ORNR-AD	Bias corrected	****	***
DWI	LMMSE-N	Multiple DWI	*****	**
	UNLM-N	Multiple DWI	*****	*****

Alternative: filtering in the complex domain (scanner) using Gaussian model.

Example: LMMSE Signal estimation

Signal estimation: the LMMSE estimator

LMSSE estimator:

$$\hat{\theta} = E\{\theta\} + \mathbf{C}_{\theta\mathbf{x}}\mathbf{C}_{\mathbf{xx}}^{-1}(\mathbf{x} - E\{\mathbf{x}\})$$

Rewriting for a 2D signal with a Rician distribution

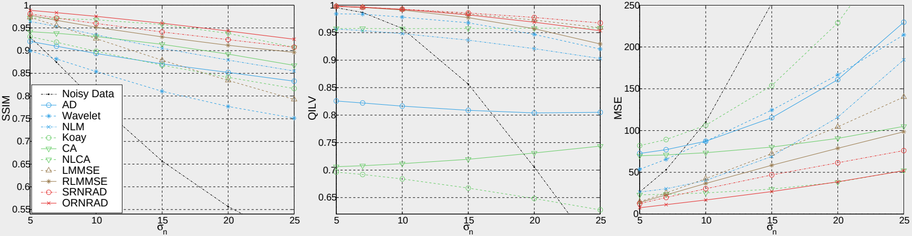
$$\widehat{A_{ij}^2} = E\{A_{ij}^2\} + \mathbf{C}_{A_{ij}^2 M_{ij}^2}\mathbf{C}_{M_{ij}^2 M_{ij}^2}^{-1}\left(\mathbf{M}_{ij}^2 - E\{\mathbf{M}_{ij}^2\}\right)$$

From here the estimator becomes:

$$\widehat{A^2(\mathbf{x})} = \langle M(\mathbf{x})^2\rangle - 2\sigma_n^2 + K(\mathbf{x})\left(M^2(\mathbf{x}) - \langle M(\mathbf{x})^2\rangle\right),$$

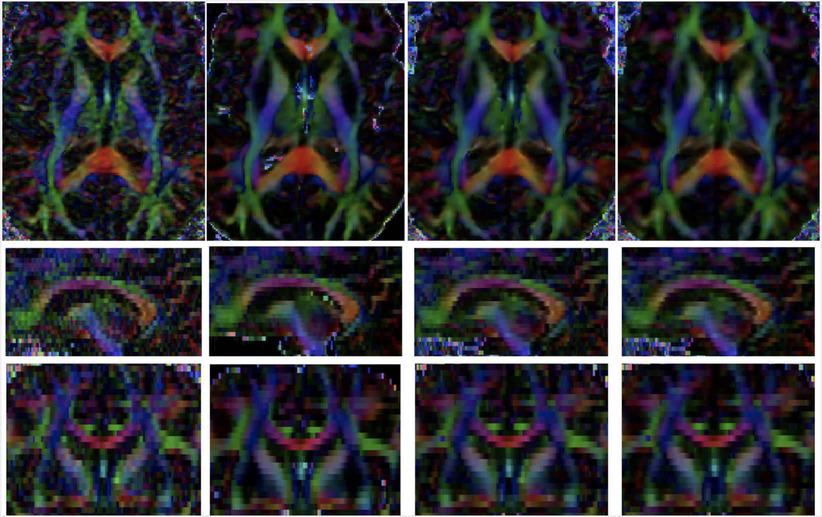
with

$$K(\mathbf{x}) = 1 - \frac{4\sigma_n^2\left(\langle M(\mathbf{x})^2\rangle - \sigma_n^2\right)}{\langle M(\mathbf{x})^4\rangle - \langle M(\mathbf{x})^2\rangle^2}.$$



	$\sigma_n = 15$			$\sigma_n = 20$			$\sigma_n = 25$		
	SSIM	QILV	MSE	SSIM	QILV	MSE	SSIM	QILV	MSE
Noise	0.6567	0.8565	252.25	0.5565	0.7053	452.33	0.4812	0.5432	710.62
AD	0.8707	0.8087	115.30	0.8520	0.8041	160.86	0.8327	0.8051	229.59
Wavelet	0.8104	0.9680	124.44	0.7770	0.9469	166.67	0.7513	0.9202	214.50
NLM	0.9051	0.9365	68.98	0.8792	0.9210	115.87	0.8550	0.9029	184.62
Koay	0.8679	0.6673	153.81	0.8412	0.6483	228.84	0.8165	0.6276	336.86
CA	0.9139	0.7197	80.21	0.8924	0.7311	90.54	0.8676	0.7436	104.99
NLCA	0.9576	0.9577	29.86	0.9379	0.9581	**38.11**	0.9071	0.9586	52.16
LMMSE	0.8789	0.9841	72.40	0.8343	0.9731	104.33	0.7924	0.9590	140.16
RLMMSE	0.9303	0.9774	58.45	0.9118	0.9572	78.70	0.8961	0.9294	98.50
SRNRAD	0.9410	**0.9859**	46.83	0.9242	**0.9777**	61.40	0.9075	**0.9677**	75.96
ORNRAD	**0.9603**	0.9824	**26.96**	**0.9432**	0.9692	38.61	**0.9251**	0.9536	**51.90**

Examples: color by orientation

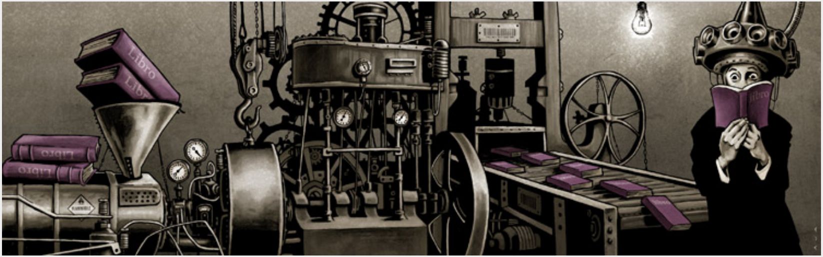


Original UNLM-5 LMMSE-1 LMMSE-15

Outline

1 Introduction
2 Signal and Noise statistical models
3 Noise filtering and signal estimation
4 Noise estimation
5 Effects on dMRI
6 Pitfalls and conclusions

Noise estimation



- Many filtering methods require an estimation of σ_n^2 (variance of noise).
- Variance of noise can be measure of quality in the data.
- Not only for filtering: Tensor estimation, segmentation methods based on the Rician distribution and fiber orientation estimators.

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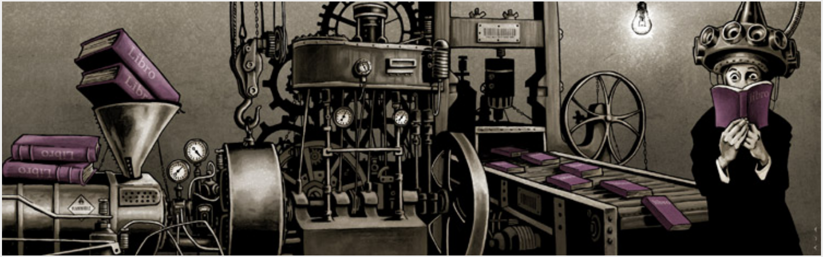

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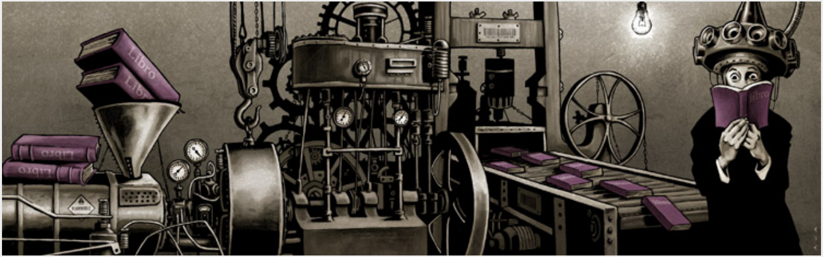

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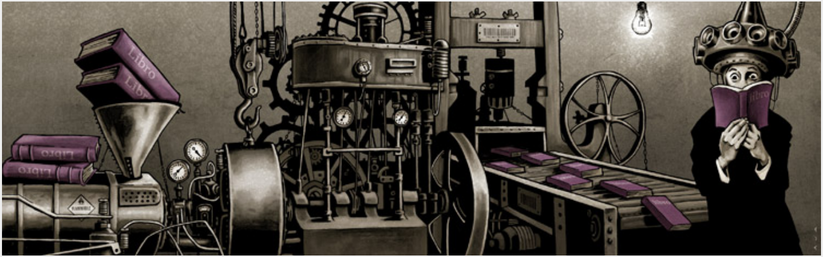

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What do we want to estimate?

Rician distribution:

$$p_M(M|A,\sigma_n) = \frac{M}{\sigma_n^2}\, e^{-\frac{M^2+A^2}{2\sigma_n^2}}\, I_0\left(\frac{AM}{\sigma_n^2}\right)\, u(M),$$

Rayleigh distribution

$$p_M(M|\sigma_n) = \frac{M}{\sigma_n^2}\, e^{-\frac{M^2}{2\sigma_n^2}}\, u(M).$$

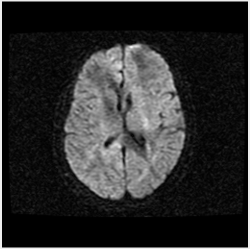
We want to estimate σ_n^2, the variance of noise in the complex **x**-space:

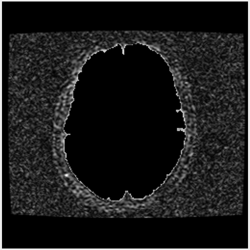
$$C(\mathbf{x}) = A(\mathbf{x}) + N(\mathbf{x};\sigma_n^2)$$

with

$$N(\mathbf{x},\sigma_n^2) = N_r(\mathbf{x};\sigma_n^2) + j\cdot N_i(\mathbf{x};\sigma_n^2)$$

How to estimate?





- We define an estimator: usually related to statistics of the image.
- E.g.: Mean of the (Rayleigh) background

$$E\{r\} = \sigma\sqrt{\frac{\pi}{2}}$$

So we can define

$$\sigma = \sqrt{\frac{2}{\pi}}E\{r\} \rightarrow \widehat{\sigma_n} = \sqrt{\frac{2}{\pi}}\langle M(\mathbf{x_B})\rangle$$

- Disadvantages: need of background segmentation; assumption of uniform background; sensitive to errors and artifacts.

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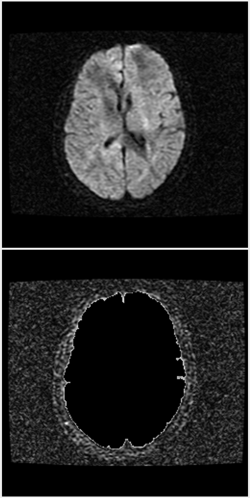

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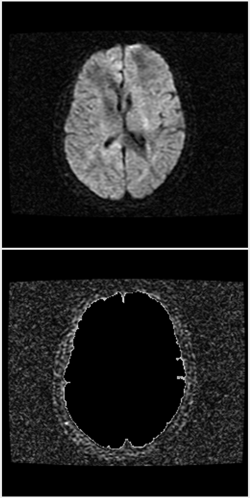

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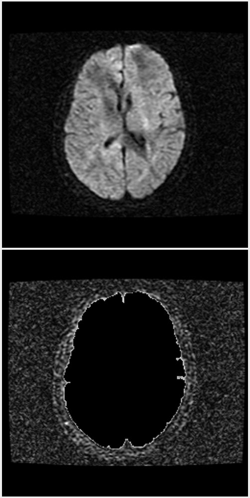

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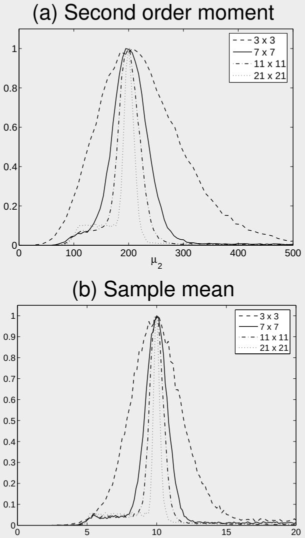


- Using the *mode* of some distribution (most probable value).
- Distribution of sample second order moment of Rayleigh data: Gamma distribution

$$S=\frac{1}{N}\sum_{i=1}^{N} R_i^2(\sigma^2)\sim\gamma\left(N,\frac{2\sigma^2}{N}\right)$$

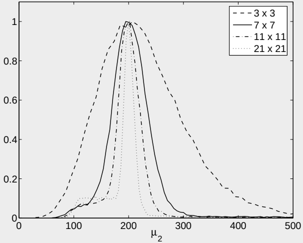
with maximum in $(N-1)/N\cdot 2\sigma_n^2$.
- So:

$$\widehat{\sigma_n^2}=\frac{N}{N-1}\cdot\frac{1}{2}\mathrm{mode}\{\langle M^2(\mathbf{x})\rangle\}\approx\frac{1}{2}\mathrm{mode}\{\langle M^2(\mathbf{x})\rangle\}$$

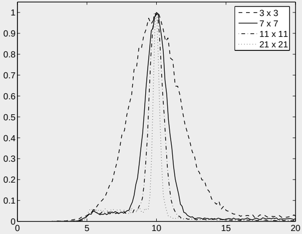
- Advantages: robust, not need of segmentation
- Disadvantages: problems to calculate the mode, *artificial* backgrounds.

How to estimate? An example

(a) Second order moment



(b) Sample mean



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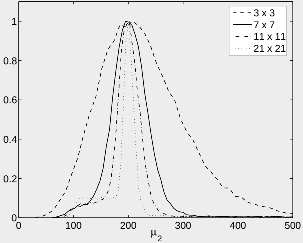

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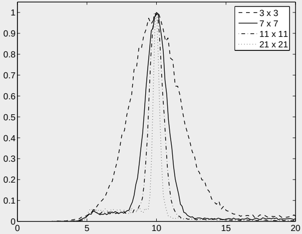

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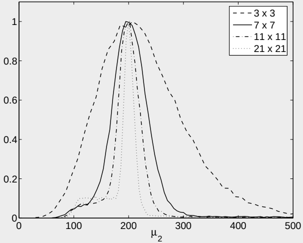

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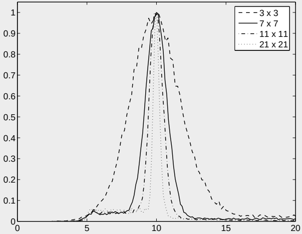

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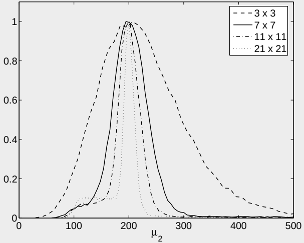

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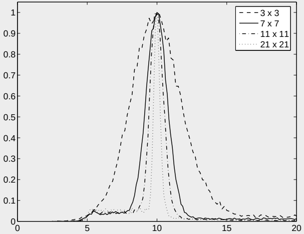

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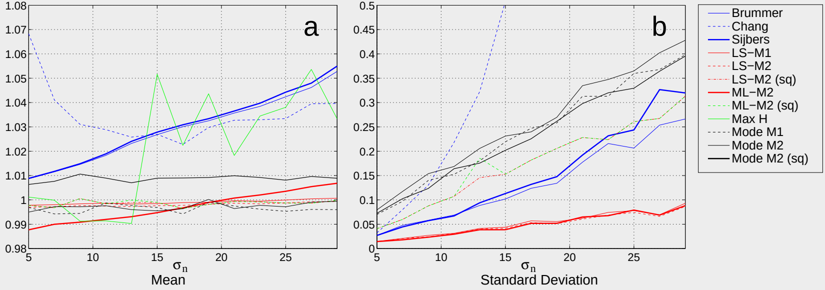

- Methods estimating from global statistic of segmented background or selected area of the background. (Rayleigh assumption)
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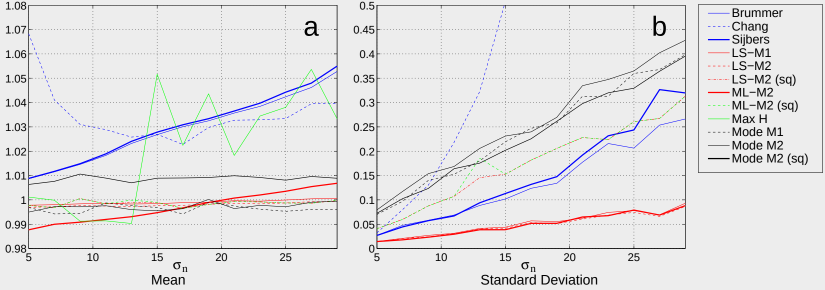

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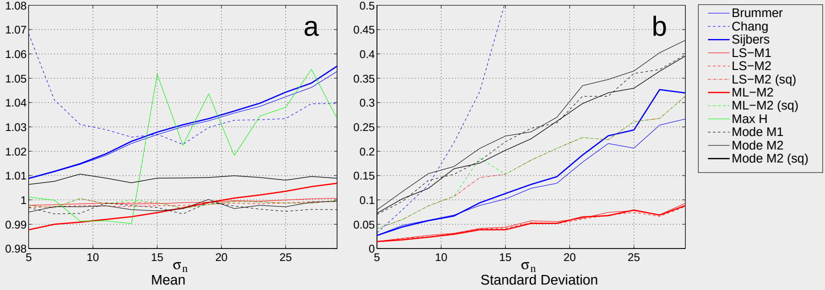

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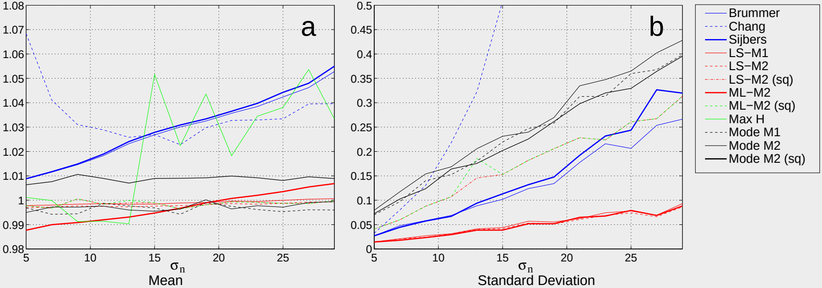

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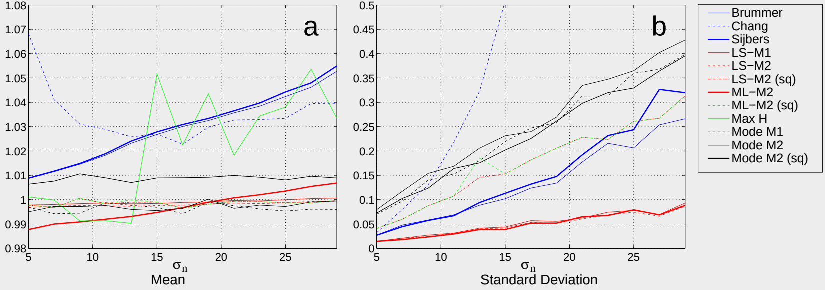

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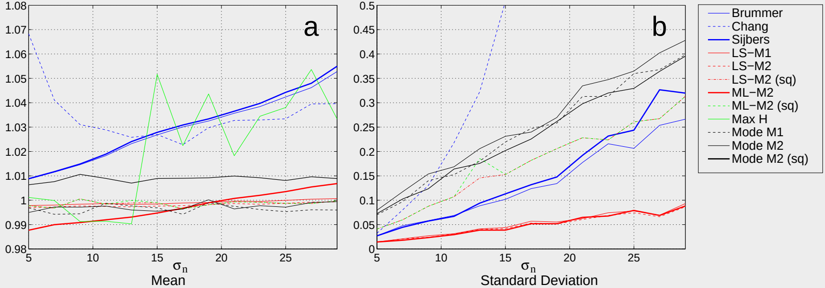

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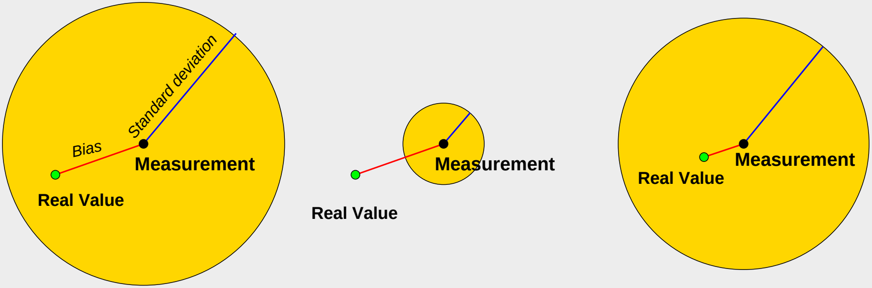
Tensor fitting based on Weighted Least Squares

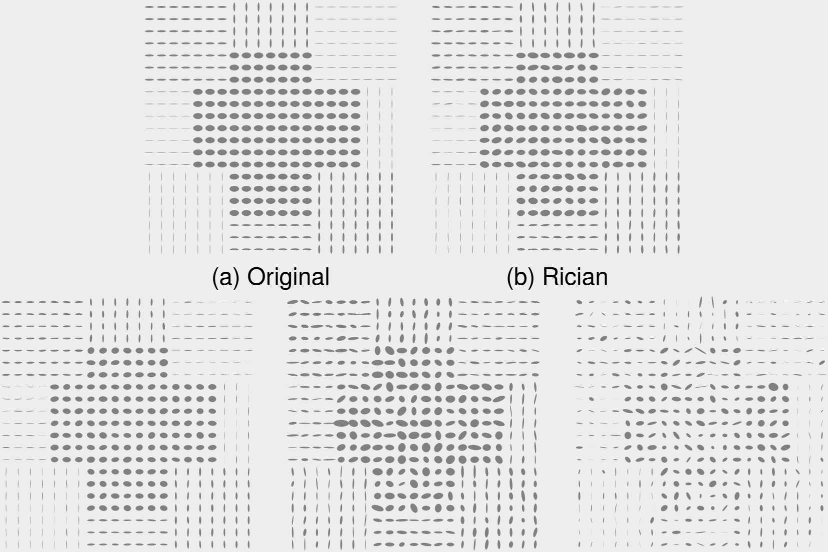
Estimation error for a simplified scenario [Tristan09]

The error (**MSE**) for multiple-coil is defined as

$$\mathrm{MSE} \simeq \underbrace{\left[\frac{K_1}{N}\left(\frac{1}{\mathrm{SNR}^2} - \frac{1}{\mathrm{SNR}^4}(3L-4)\right)\right]}_{\mathrm{Var(estimation)}} + \underbrace{\left[\frac{1}{\mathrm{SNR}^4}3(L-1)^2\right]}_{\mathrm{bias}^2\mathrm{(estimation)}}$$



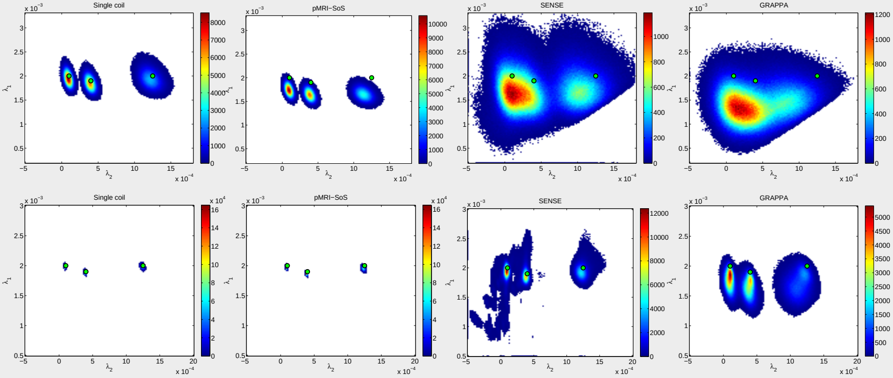
Synthetic experiments



(a) Original (b) Rician

(c) pMRI-SoS (d) SENSE (e) GRAPPA

Estimation tensor: Synthetic experiments



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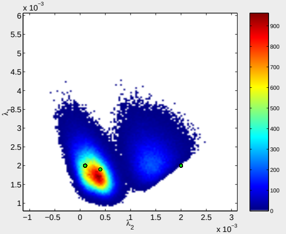
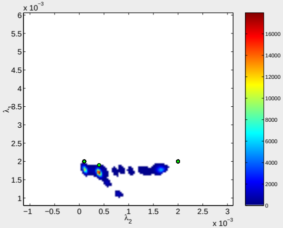
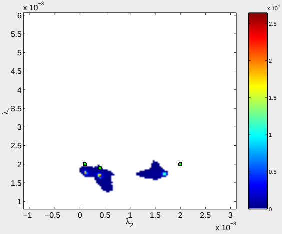


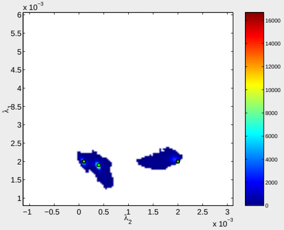
(a) Noisy



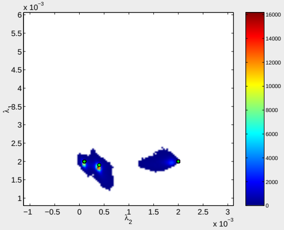
(b) Gaussian



(c) Wiener

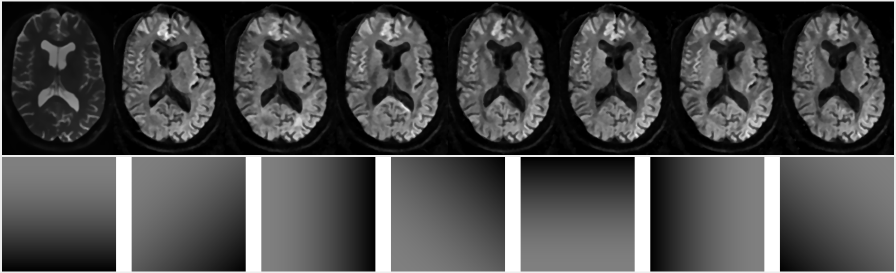


(d) LMMSE



(e) RLMMSE (5)

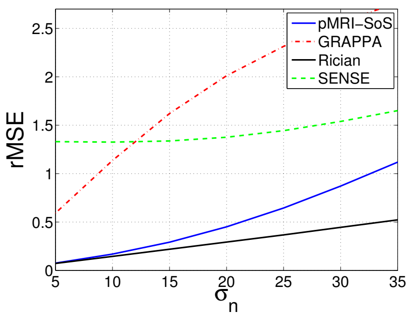
Realistic DWI phantom

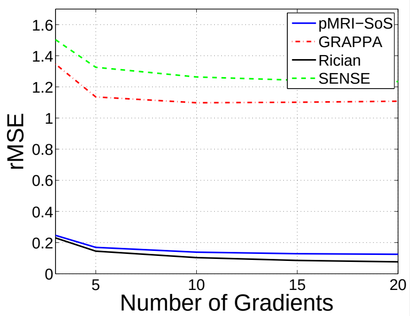


A realistic DWI phantom is used, [Tristan09b]. A $256 \times 256 \times 81$ volume, spatial resolution of 1mm$\times$ 1mm$\times$ 1.7mm, 15 gradient directions and 1 baseline.

[Tristan09b] A. Tristán-Vega and S. Aja-Fernández, “Design and construction of a realistic DWI phantom for filtering performance assessment,” in *MICCAI 2009*, 2009.

Realistic DWI phantom

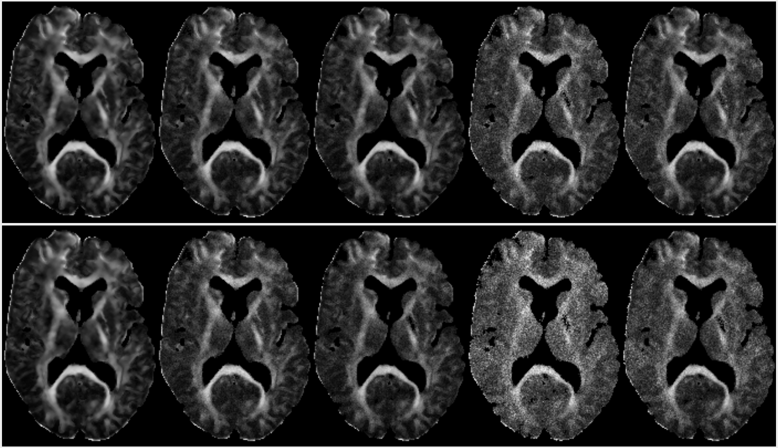




rMSE of the tensor estimation for (Left) different σ_n values (and 5 gradients); (Right) different number of gradients (and $\sigma_n = 10$)

$$\text{rMSE}(x) = \frac{\sqrt{(\widehat{\lambda_1}(x) - \lambda_1(x))^2 + (\widehat{\lambda_2}(x) - \lambda_2(x))^2 + (\widehat{\lambda_3}(x) - \lambda_3(x))^2}}{\lambda_1(x)}$$

Realistic DWI phantom



Fractional Anisotropy. From left to right: Original non-noisy data; Rician case; pMRI-SoS case; SENSE case; GRAPPA case. Top row $\sigma_n = 10$ (average SNR in gray matter in the gradient images 40). Low row: $\sigma_n = 35$ (average SNR in gray matter in the gradient images 11.4).

Outline

1. Introduction
2. Signal and Noise statistical models
3. Noise filtering and signal estimation
4. Noise estimation
5. Effects on dMRI
6. Pitfalls and conclusions

Pitfalls and conclusions

Pitfalls

- Acquisition: reduced **k**-space and EPI introduces non-linearity that make the signal differs from model.
- Correlations must be taking into account.
- Parallel acquisition: Non-stationary model. Is noise estimation possible? Has it any meaning?

Conclusions

- Noise affects not only the visual quality but the estimation of diffusion parameters.
- Knowing the underlying noise model helps to better filtering.
- Proper noise estimation improves signal estimation (and noise filtering).
- Better to filter BEFORE estimation.

References: 3 papers to start

- Aja-Fernández, S., Tristán-Vega, A., Alberola-López, C., 2009. *Noise estimation in single and multiple coil MR data based on statistical models*. **Magn. Reson. Imag**. 27, 1397–1409.
- Tristán-Vega, A., Aja-Fernández, S., 2010. *DWI filtering using joint information for DTI and HARDI*. **Med. Imag. Anal**. 14 (2), 205 – 218.
- Aja-Fernández, S., Tristán-Vega, A., de-la Higuera, P. C., 2010. *DWI acquisition schemes and diffusion tensor estimation: A simulation–based study*. In: Proc. of the 32nd Annual International Conference of the IEEE Engineering in Medicine and Biology Society (**EMBC'2010**). Buenos Aires, Argentina.

Thanks for you attention!

Noise estimation and diffusion signal reconstruction: From cradle to parallel imaging

What type of noise ‘infects’ the data and by filtering it out are we (black) magically creating something new?

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